Contents lists available at ScienceDirect

Signal Processing



journal homepage: www.elsevier.com/locate/sigpro

Fast communication

Color image canonical correlation analysis for face feature extraction and recognition

Xiaoyuan Jing^{a,d,*}, Sheng Li^a, Chao Lan^a, David Zhang^b, Jingyu Yang^c, Qian Liu^a

^a College of Automation, Nanjing University of Posts and Telecommunications, PR China

^b Department of Computing, Hong Kong Polytechnic University, Hong Kong

^c College of Computer Science, Nanjing University of Science and Technology, PR China

^d State Key Laboratory for Novel Software Technology, Nanjing University, PR China

ARTICLE INFO

Article history: Received 18 July 2010 Received in revised form 29 December 2010 Accepted 21 February 2011 Available online 24 February 2011

Keywords: Canonical correlation analysis (CCA) Color image CCA (CICCA) Feature extraction Color face recognition.

ABSTRACT

Canonical correlation analysis (CCA) is a powerful statistical analysis technique, which can extract canonical correlated features from two data sets. However, it cannot be directly used for color images that are usually represented by three data sets, i.e., red, green and blue components. Current multi-set CCA (mCCA) methods, on the other hand, can only provide the iterative solutions, not the analytical solutions, when processing multiple data sets. In this paper, we develop the CCA technique and propose a color image CCA (CICCA) approach, which can extract canonical correlated features from three color components and provide the analytical solution. We show the mathematical model of CICCA, prove that CICCA can be cast as solving three eigen-equations, and present the realization algorithm of CICCA. Experimental results on the AR and FRGC-2 public color face image databases demonstrate that CICCA outperforms several representative color face recognition methods.

© 2011 Elsevier B.V. All rights reserved.

1. Introduction

The data of color images are two times more than those of grayscale images and thus color images can provide more useful information than grayscale images yet may contain certain amount of redundant information. Therefore, how to effectively exploit color information to improve classification performance while reducing redundancy emerges as an important task in color image recognition. In literature some methods are dedicated to tackle this issue. Yang and Liu [1,2] presented an extended general color image discriminant (Extended GCID) algorithm that produces three groups of weights to fuse color components and then extracts discriminant features from the fused components. Liu [3] presented the

E-mail addresses: jingxy_2000@126.com, jingxy@njupt.edu.cn (X. Jing).

0165-1684/\$ - see front matter \circledcirc 2011 Elsevier B.V. All rights reserved. doi:10.1016/j.sigpro.2011.02.016

uncorrelated color space (UCS), the independent color space (ICS), and the discriminating color space (DCS) methods for face recognition. UCS applies the principal component analysis (PCA) [4] to reduce the correlations between color components. ICS assumes that each color image is defined by three independent source images that can be derived by a blind source separation procedure, such as the independent component analysis (ICA) [5]. And DCS applies the discriminant analysis technique [6-8] to define three new component images that are effective for recognition. Yang et al. [9] transformed RGB space to HSV color space, and then employed the Hue (H) and Saturation (S) components to perform PCA transform. Jones and Abbott [10] employed the hypercomplex form to fuse the red, green and blue color components and then extracted features. Yang et al. [11] presented two color space normalization techniques. Current representative color face recognition methods (including Extended GCID, UCS, ICS, DCS, etc.) first transform the original RGB space to new color spaces, and then employ commonly used



^{*} Corresponding author.

linear discriminant analysis techniques to extract features and do classification. These recognition methods reduce the correlations between three color components in the pixel level. In contrast, in this paper, we exploit color information in another way, that is, we develop a novel feature extraction technique and reduce correlations among different color components in the feature level.

Canonical correlation analysis (CCA) is an effective statistical analysis technique, which is first proposed by Hotelling [12] as a way to measure the mutual relationship between two multidimensional data sets. CCA seeks the transformation vectors and represents a high-dimensional relationship between two data sets with a few pairs of canonical variables. It has been shown that the transformation vectors of CCA can be obtained by solving the eigen-problem, that is, the analytic solution of CCA can be obtained [13]. The CCA technique has been widely studied in several fields such as signal processing [14], computer vision [15], and pattern recognition [16]. Since it can extract image canonical correlated features, it has been successfully applied to image recognition [17].

Multi-set canonical correlations analysis (mCCA) [18] extends CCA technique and aims at analyzing the correlations between more (than two) data sets. Li et al. [19] pointed out in his work that mCCA cannot be solved as an eigen-problem due to its multiple random vectors. Therefore, current mCCA methods are solved in an iterative way: Via et al. [20] proposed a neural network model and the corresponding adaptive algorithm to realize mCCA for signal processing; Li et al. [19] and Correa et al. [21] presented the iterative solutions of mCCA for joint blind source separation and brain imaging data fusion; Hasan [22] proposed an mCCA dynamical systems which iteratively compute the multi-set canonical correlations and canonical variates. The mCCA methods cannot be employed for the image recognition task, due to its lack of analytic solutions and large computational burden.

1.1. Motivation and contribution

In this paper, we attempt to analyze the canonical correlations for color images and extract effective features for recognition. However, traditional CCA cannot be applied directly to this problem at hand, since it can only process two data sets while we have three data sets (i.e., red, green and blue components) to process. Current mCCA methods, on the other hand, solve the problem in an iterative manner, which may be computational expensive, and fail to provide the analytical solutions. In this paper, we propose a color image CCA (CICCA) approach for feature extraction and recognition, which can provide the analytic solution and extract effective features. We first describe the theoretical foundations of CICCA and present an analytic solution to it. We show that the proposed CICCA can be cast as solving three eigen-equations. Next, we explain how to preprocess the color components. And then we present the realization algorithm of CICCA. Experimental results on the AR and FRGC-2 public color face image databases demonstrate that the proposed approach outperforms several representative color face recognition methods.

1.2. Organization

The remainder of this paper is organized as follows. In Section 2, we describe the theoretical foundation of the proposed CICCA approach. In Section 3, we provide the realization algorithm of CICCA. In Section 4, we show the experiments on the AR and FRGC-2 public color face image databases. Finally, conclusions are drawn in Section 5.

2. Theoretical foundation of color image canonical correlation analysis (CICCA)

In this section, we describe the theoretical analysis of the proposed CICCA approach and present the analytical solution of CICCA.

Given *N* pairs of data (x_i, y_i, z_i) , $i = 1, \dots, N$, $x_i \in \mathbb{R}^p$, $y_i \in \mathbb{R}^q$, $z_i \in \mathbb{R}^s$. We try to find three projection vectors φ_x , φ_y and φ_z that can maximize the correlations between $\varphi_x^T(x_i - \overline{x})$, $\varphi_y^T(y_i - \overline{y})$ and $\varphi_z^T(z_i - \overline{z})$, respectively, thus we can acquire the typically correlative features of three input data sets. The correlations r_1 , r_2 , r_3 are defined as

$$r_1 = \frac{\varphi_x^T X Y^T \varphi_y}{\sqrt{\varphi_x^T X X^T \varphi_x} \sqrt{\varphi_y^T Y Y^T \varphi_y}},\tag{1}$$

$$r_2 = \frac{\varphi_y^T Y Z^T \varphi_z}{\sqrt{\varphi_y^T Y Y^T \varphi_y} \sqrt{\varphi_z^T Z Z^T \varphi_z}},$$
(2)

$$r_{3} = \frac{\varphi_{z}^{T} Z X^{T} \varphi_{x}}{\sqrt{\varphi_{z}^{T} Z Z^{T} \varphi_{z}} \sqrt{\varphi_{x}^{T} X X^{T} \varphi_{x}}}$$
(3)

where $X = [x_1 - \overline{x}, \dots, x_N - \overline{x}]$, $Y = [y_1 - \overline{y}, \dots, y_N - \overline{y}]$, $Z = [z_1 - \overline{z}, \dots, z_N - \overline{z}]$, \overline{x} , \overline{y} and \overline{z} are the mean vectors of X, Y and Z, respectively. To get the maximum values of r_1, r_2, r_3 , Eqs. (1), (2) and (3) are transformed to the following optimization problem:

$$\max \ \varphi_x^T X Y^T \varphi_y + \varphi_y^T Y Z^T \varphi_z + \varphi_z^T Z X^T \varphi_x,$$

s.t.
$$\varphi_x^T X X^T \varphi_x = 1, \ \varphi_y^T Y Y^T \varphi_y = 1, \ \varphi_z^T Z Z^T \varphi_z = 1.$$
(4)

Eq. (4) makes three input data sets typically correlated. To find the analytic solution of Eq. (4), we give a theorem and its proof as follows:

Theorem 1. The solution of Eq. (4) is equivalent to solving the following three eigen-equations:

$$\begin{cases} XY^{T}(YY^{T})^{-1}YZ^{T}(ZZ^{T})^{-1}ZX^{T}\varphi_{x} = \lambda XX^{T}\varphi_{x} \\ YZ^{T}(ZZ^{T})^{-1}ZX^{T}(XX^{T})^{-1}XY^{T}\varphi_{y} = \lambda YY^{T}\varphi_{y} \\ ZX^{T}(XX^{T})^{-1}XY^{T}(YY^{T})^{-1}YZ^{T}\varphi_{z} = \lambda ZZ^{T}\varphi_{z} \end{cases}$$
(5)

The proof is given in Appendix.

3. Realization algorithm of CICCA

This section explains how to apply CICCA to color images for feature extraction and recognition.

3.1. Preprocessing of color components

Color images are usually expressed by three color components, namely, red (R), green (G) and blue (B).

In this paper, we use the preprocessed R, G and B color components as three input data sets of CICCA. Three main reasons for the preprocessing phrase are presented below.

First, we give the reason of preprocessing. The original R, G and B color components are generally high dimensional. If they are directly employed to CICCA, it is easy to cause the singularity problem of matrices, which makes the solutions of inverse matrices in Eq. (5) difficult. In addition, these data have redundancy information that is not helpful to feature extraction and recognition. Hence, we need to reduce the dimensions of R, G and B color components before using CICCA.

Second, we provide the preprocessing method. The original R, G and B color components are greatly correlated data sets. There are strong correlations among R, G and B color components. If we deal with these components by using the same dimensionality reduction method, they will also be greatly correlated after dimensionality reduction. Thus the features extracted by CICCA are greatly correlated and many of these features are redundant. This redundancy will reduce the classification performance of extracted features.

Third, the CCA technique typically extracts the canonically correlated features from two input data sets with comparably large difference [12–17].

Consequently, in this paper, we preprocess R, G and B color components using three different conventional dimension reduction (DM) methods, namely, PCA [4], LDA [6] and MSDDA [25]. To show the effectiveness of CICCA, we test all combinations of applying three DM methods to R, G and B color components in the experiment, where one DM method is only used for one color component.

3.2. Realization algorithm description

Let X_R , X_G and X_B denote the data sets consisting of R, G and B components, respectively. The CICCA approach can be realized as follows:

Step 1. Preprocess X_R , X_G and X_B by three methods (including PCA, LDA and MSDDA), and obtain three new data sets Y_R , Y_G and Y_B .

Step 2. Perform CICCA on Y_R , Y_G and Y_B , and calculate three projection transforms V_R , V_G and V_B using Eq. (5).

Step 3. Construct a new overall data set Z by

$$Z = \left[Y_R^T V_R, Y_G^T V_G, Y_B^T V_B\right]^T.$$
(6)

Step 4. Use the nearest neighbor classifier with the cosine distance to classify *Z*.

4. Experimental results

In this section, we compare the classification performance of the proposed CICCA approach with several representative color face recognition methods on AR and FRGC-2 color face image databases. The AR color face database [23] contains 102 individuals with each person contributing 26 images. The FRGC-2 database [24] used in the experiment contains 100 individuals, each 24 images. We crop every image to the size of 60×60 , and show images of one subject from AR and FRGC-2 databases in Figs. 1 and 2, respectively. The experiments are carried out on a T7250 2.0GHz computer with a 2 GB RAM and tested on the Matlab 7.8.0 platform.

In the experiments, we randomly select 6 sample images per person for training, use the remainder for testing and run all recognition methods for 30 times. In order to prove the effectiveness of CICCA, we test all six combinations of applying three preprocessing methods, that is, PCA [4], LDA [6] and MSDDA [25], to R, G and B color components, where one preprocessing method is only used for one color component. For PCA, LDA, MSDDA and CICCA methods, we employ the same classifier, i.e., the nearest neighbor classifier with cosine distance, to do classification.

Figs. 3 and 4 show the recognition rates of all six combinations of applying three preprocessing methods, i.e., PCA, LDA and MSDDA to R, G and B color components and the corresponding CICCA approach on the AR and FRGC-2 databases, respectively. For example, Fig. 3 (a) shows the recognition rates of PCA-R, LDA-G, MSDDA-B and corresponding CICCA on the AR database, where PCA-R denotes applying PCA to R component, LDA-G denotes applying LDA to G component; and three input data sets of CICCA include R component preprocessed by PCA,



Fig. 1. Demo images of one individual on AR database.



Fig. 2. Demo images of one individual on FRGC-2 database.



Fig. 3. Recognition rates of all six combinations of applying PCA, LDA and MSDDA to R, G and B color components, and rates of corresponding CICCA approach on AR database.



Fig. 4. Recognition rates of all six combinations of applying PCA, LDA and MSDDA to R, G and B color components, and rates of corresponding CICCA approach on FRGC-2 database.

G component preprocessed by LDA and B component preprocessed by MSDDA.

Table 1 shows the classification performance of PCA, LDA and MSDDA preprocessing R, G and B color components on the AR and FRGC-2 databases. Meanwhile, we list the classification results of CICCA. Figs. 3 and 4 show the recognition results of different combinations, that is, (i) PCA-R, LDA-G, MSDDA-B and the corresponding CICCA; (ii) PCA-R, LDA-B, MSDDA-G and CICCA; (iii) PCA-G, LDA-R, MSDDA-B and CICCA; (iv) PCA-G, LDA-B, MSDDA-R and CICCA; (v) PCA-B, LDA-G, MSDDA-R and CICCA; (v) PCA-B, LDA-C, MSDDA-C, mean combinations lead to slight variations (from 83.72% to 85.38% on the AR database, from 86.88% to 88.32% on the FRGC database)

in the classification results of CICCA. In Table 1, we show the lowest and highest results of CICCA among the results of all six combinations.

By constructing two appropriate input data sets, the CCA technique [19] might obtain preferable recognition effect. Similarly, the proposed CICCA approach can select most appropriate three input data sets (among six combinations in Figs. 3 and 4) to enhance recognition effect. Hence, we use the highest classification performance in Table 1 to express the classification result of CICCA.

Figs. 5 and 6 show the recognition rates of the proposed CICCA approach and several representative grayscale and color face recognition methods including Extended GCID [1], DCS [3], Gray-PCA, Gray-LDA, Gray-MSDDA, Color PCA (CPCA), Color LDA (CLDA) and Color MSDDA (CMSDDA) on the AR and FRGC-2 databases, respectively. The CPCA, CLDA and CMSDDA methods are the extensions of the grayscale PCA [4], LDA [6] and MSDDA [25] methods. They

Table 1

Classification performance of PCA, LDA, MSDDA and CICCA on AR and FRGC-2 color databases.

Method	Mean and standard deviation of recognition rates (%)	
	AR	FRGC-2
PCA-R PCA-G PCA-B LDA-R LDA-G	$\begin{array}{c} 68.43 \pm 6.08 \\ 69.32 \pm 7.36 \\ 77.03 \pm 7.52 \\ 77.34 \pm 6.45 \\ 78.16 \pm 6.51 \\ 20.25 = 7.52 \end{array}$	$\begin{array}{c} 63.63 \pm 5.53 \\ 59.81 \pm 5.87 \\ 58.02 \pm 5.74 \\ 82.53 \pm 3.79 \\ 83.95 \pm 3.61 \\ \end{array}$
LDA-B MSDDA-R MSDDA-G MSDDA-B <i>CICCA</i> Lowest Highest	76.26 ± 7.50 64.03 ± 5.47 65.47 ± 7.02 76.22 ± 7.71 83.72 ± 6.98 85.38 ± 6.30	$\begin{array}{c} 83.94 \pm 3.87 \\ 63.53 \pm 5.54 \\ 59.69 \pm 5.85 \\ 58.02 \pm 5.73 \\ \end{array}$ $\begin{array}{c} 86.88 \pm 3.64 \\ 88.32 \pm 3.52 \end{array}$

combine the red, green and blue component vectors of each sample into a vector and then extract features. According to Figs. 5 and 6, CICCA outperforms other face recognition methods.

Table 2 shows the means and standard deviations of recognition rates of CICCA and several representative

Table 2

Classification performances of Color face recognition methods on AR and FRGC-2 databases.

Method	Mean and standard deviation of recognition rates (%)		
	AR	FRGC-2	
Gray-PCA	70.90 ± 7.06	60.87 ± 5.94	
Gray-LDA	78.78 ± 6.64	83.39 ± 3.52	
Gray-MSDDA	67.02 ± 7.12	60.73 ± 5.90	
CPCA	73.00 ± 7.18	61.03 ± 5.85	
CLDA	80.45 ± 6.04	84.42 ± 3.51	
CMSDDA	68.76 ± 6.64	57.95 ± 5.86	
Extended GCID	82.28 ± 6.08	85.60 ± 3.27	
DCS	81.24 ± 6.05	85.02 ± 3.29	
CICCA	85.38 ± 6.30	88.32 ± 3.52	



Fig. 5. Recognition rates of CICCA and other representative face recognition methods on AR database.



Fig. 6. Recognition rates of CICCA and other representative face recognition methods on FRGC-2 database.



Fig. 7. The FRR and FAR of CICCA on two face databases. (a) AR database and (b) FRGC-2 database.



Fig. 8. ROC curves of CICCA and compared methods on two databases. (a) AR database and (b) FRGC-2 database.

grayscale and color face recognition methods on the AR and FRGC-2 color face image databases. Compared with other methods, the proposed CICCA approach improves the average

recognition rates at least by 3.1% (=85.38%-82.28%) and 2.72% (=88.32%-85.60%) on the AR and FRGC-2 databases, respectively.

Table 3 Time complexity and average computing time (s) of CICCA and compared methods.

Method	Time complexity	Average computing time (s)
CICCA	$O(N^3)$	20.89
Extended GCID	$O(kN^3)$	35.77
DCS	$O(N^3)$	35.46
Color PCA	$O(N^3)$	19.81
Color LDA	$O(N^3)$	18.74
Color MSDDA	$O(N^3)$	19.38

N is the total number of training samples and k is the number of iterations of Extended GCID.

To further analyze the performance of the proposed approach, we evaluate the false reject rate (FRR) and false accept rate (FAR) of CICCA, and show the results on two databases in Fig. 7.

The ROC curves of CICCA and compared methods are given in Fig. 8. It can be seen that CICCA attains comparatively low equal error rate (EER), and thus is an effective face recognition approach.

In addition, Table 3 shows the time complexity and average computing time of CICCA and other color face recognition methods. Table 3 shows that, compared with Extended GCID and DCS, the proposed CICCA approach consumes less time.

5. Conclusions

In this paper, we develop the canonical correlation analysis (CCA) technique and propose a color image canonical correlation analysis (CICCA) approach for feature extraction and recognition. We derive the analytical solution of CICCA and present its realization algorithm. Experiments on AR and FRGC-2 color face databases show that, CICCA achieves better recognition results than several representative color face recognition methods, which improves the average recognition rates by at least 3.1% and 2.72% in contrast with compared methods on AR and FRGC-2 databases, respectively. Accordingly, the proposed CICCA approach is well-suited for color face recognition.

Acknowledgements

The work described in this paper was fully supported by the NSFC under Project No. 60772059, 61073113, the New Century Excellent Talents of Education Ministry under Project No. NCET-09-0162, the Doctoral Foundation of Education Ministry under Project No. 20093223110001, the Qing-Lan Engineering Academic Leader of Jiangsu Province, the Foundation of Jiangsu Province Universities under Project No. 09KJB510011, the Foundation of NJUPT under Project No. NY207027, NY208051.

Appendix

The proof of Theorem 1

Proof. By using the Lagrange multipliers, Eq. (4) can be reformulated as:

$$L = \varphi_x^T X Y^T \varphi_y + \varphi_y^T Y Z^T \varphi_z + \varphi_z^T Z X^T \varphi_x + \lambda_x (1 - \varphi_x^T X X^T \varphi_x) + \lambda_y (1 - \varphi_y^T Y Y^T \varphi_y) + \lambda_z (1 - \varphi_z^T Z Z^T \varphi_z),$$
(7)

where λ_x , λ_y and λ_z are the Lagrange multipliers.

Separately set the partial derivative $\partial(L)/\partial(\varphi_x)$, $\partial(L)/\partial(\varphi_y)$ and $\partial(L)/\partial(\varphi_z)$ to zero, we get:

$$XY^{T}\varphi_{y} + XZ^{T}\varphi_{z} - 2\lambda_{x}XX^{T}\varphi_{x} = 0,$$
(8)

$$YX^{T}\varphi_{x} + YZ^{T}\varphi_{z} - 2\lambda_{y}YY^{T}\varphi_{y} = 0,$$
(9)

$$ZY^{T}\varphi_{y} + ZX^{T}\varphi_{x} - 2\lambda_{z}ZZ^{T}\varphi_{z} = 0.$$
⁽¹⁰⁾

Separately left multiplying both sides of Eqs. (8)–(10) by φ_{x}^{T} , φ_{y}^{T} and φ_{z}^{T} , we have

$$\varphi_x^T X Y^T \varphi_y + \varphi_x^T X Z^T \varphi_z = 2\lambda_x, \tag{11}$$

$$\varphi_y^T Y X^T \varphi_x + \varphi_y^T Y Z^T \varphi_z = 2\lambda_y, \tag{12}$$

$$\varphi_z^T Z Y^T \varphi_y + \varphi_z^T Z X^T \varphi_x = 2\lambda_z.$$
(13)

Separately substituting Eqs. (11)–(13) into Eq. (7), we have

$$L_{1} = \varphi_{y}^{T} Y Z^{T} \varphi_{z} + 2\lambda_{x} + \lambda_{x} (1 - \varphi_{x}^{T} X X^{T} \varphi_{x}) + \lambda_{y} (1 - \varphi_{y}^{T} Y Y^{T} \varphi_{y}) + \lambda_{z} (1 - \varphi_{z}^{T} Z Z^{T} \varphi_{z}),$$
(14)

$$L_{2} = \varphi_{z}^{T} Z X^{T} \varphi_{x} + 2\lambda_{y} + \lambda_{x} (1 - \varphi_{x}^{T} X X^{T} \varphi_{x}) + \lambda_{y} (1 - \varphi_{y}^{T} Y Y^{T} \varphi_{y}) + \lambda_{z} (1 - \varphi_{z}^{T} Z Z^{T} \varphi_{z}),$$
(15)

$$L_{3} = \varphi_{x}^{T} X Y^{T} \varphi_{y} + 2\lambda_{z} + \lambda_{x} (1 - \varphi_{x}^{T} X X^{T} \varphi_{x}) + \lambda_{y} (1 - \varphi_{y}^{T} Y Y^{T} \varphi_{y}) + \lambda_{z} (1 - \varphi_{z}^{T} Z Z^{T} \varphi_{z}).$$
(16)

We set $\partial(L_1)/\partial(\varphi_y)$, $\partial(L_2)/\partial(\varphi_z)$ and $\partial(L_3)/\partial(\varphi_x)$ to zero, and obtain

$$YZ^T \varphi_z - 2\lambda_y YY^T \varphi_y = 0, \tag{17}$$

$$ZX^{T}\varphi_{x}-2\lambda_{z}ZZ^{T}\varphi_{z}=0, \qquad (18)$$

$$XY^{T}\varphi_{y} - 2\lambda_{x}XX^{T}\varphi_{x} = 0.$$
(19)
Eq. (18) can be rewritten as

Eq. (18) call be rewritten as

$$\varphi_z = \frac{1}{2\lambda_z} (ZZ^T)^{-1} ZX^T \varphi_x.$$
⁽²⁰⁾

Substituting Eq. (20) into Eq. (17), we have

$$\varphi_{y} = \frac{1}{4\lambda_{y}\lambda_{z}} Y Y^{T} Y Z^{T} (ZZ^{T})^{-1} Z X^{T} \varphi_{x}.$$
(21)

Substituting Eq. (21) into Eq. (19), we have

$$XY^{T}(YY^{T})^{-1}YZ^{T}(ZZ^{T})^{-1}ZX^{T}\varphi_{x} = 8\lambda_{x}\lambda_{y}\lambda_{z}XX^{T}\varphi_{x}, \qquad (22)$$

In this manner, we can also get

$$YZ^{T}(ZZ^{T})^{-1}ZX^{T}(XX^{T})^{-1}XY^{T}\varphi_{y} = 8\lambda_{x}\lambda_{y}\lambda_{z}YY^{T}\varphi_{y},$$
(23)

$$ZX^{T}(XX^{T})^{-1}XY^{T}(YY^{T})^{-1}YZ^{T}\varphi_{z} = 8\lambda_{x}\lambda_{y}\lambda_{z}ZZ^{T}\varphi_{z}.$$
(24)

Set
$$\lambda = 8\lambda_x \lambda_y \lambda_z$$
, and then we have

$$XY^{T}(YY^{T})^{-1}YZ^{T}(ZZ^{T})^{-1}ZX^{T}\varphi_{x} = \lambda XX^{T}\varphi_{x},$$
(25)

$$YZ^{T}(ZZ^{T})^{-1}ZX^{T}(XX^{T})^{-1}XY^{T}\varphi_{y} = \lambda YY^{T}\varphi_{y}, \qquad (26)$$

$$ZX^{T}(XX^{T})^{-1}XY^{T}(YY^{T})^{-1}YZ^{T}\varphi_{z} = \lambda ZZ^{T}\varphi_{z}. \qquad \Box \qquad (27)$$

References

- J. Yang, C. Liu, Color image discriminant models and algorithms for face recognition, IEEE Transactions on Neural Networks 19 (12) (2008) 2088–2098.
- [2] J. Yang, C. Liu, A general discriminant model for color face recognition, International Conference on Computer Vision (2007) 1–6.
- [3] C. Liu, Learning the uncorrelated, independent, and discriminating color spaces for face recognition, IEEE Transactions on Information Forensics and Security 3 (2) (2008) 213–222.
- [4] M. Turk, A. Pentland, Eigenfaces for recognition, Journal of Cognitive Neuroscience 3 (1) (1991) 71–86.
- [5] P. Comon, Independent component analysis, a new concept?, Signal Processing 36 (3) (1994) 287–314
- [6] P. Belhumeur, N.J. Hespanda, D. Kiregeman, Eigenfaces vs. Fisherfaces: recognition using class specific linear projection, IEEE Transactions on Pattern Analysis and Machine Intelligence 19 (7) (1997) 711–720.
- [7] W.M. Zuo, H.Z. Zhang, D. Zhang, K.Q. Wang, Post-processed LDA for face and palmprint recognition: what is the rationale, Signal Processing 90 (8) (2010) 2344–2352.
- [8] X.Y. Jing, D. Zhang, Y.Y. Tang, An improved LDA approach, IEEE Transactions on Systems Man and Cybernetics Part B: Cybernetics 34 (5) (2004) 1942–1951.
- [9] J. Yang, D. Zhang, Y. Xu, J.Y. Yang, Recognize color face images using complex Eigenfaces, International Conference on Biometrics (2006) 64–68.
- [10] C. Jones, A. Abbott, Color face recognition by hypercomplex Gabor analysis, International Conference on Automatic Face and Gesture Recognition (2006) 126–131.

- [11] J. Yang, C. Liu, L. Zhang, Color space normalization: enhancing the discriminating power of color spaces for face recognition, Pattern Recognition 43 (4) (2010) 1454–1466.
- [12] H. Hotelling, Relations between two sets of variates, Biometrika 28 (3-4) (1936) 321-377.
- [13] T.W. Anderson, An Introduction to Multivariate Statistical Analysis, Wiley, New York, 1984.
- [14] S. Yu, Direct blind channel equalization via the programmable canonical correlation analysis, Signal Processing 81 (8) (2001) 1715–1724.
- [15] M. Loog, B. Ginneken, R.P.W. Duin, Dimensionality reduction by canonical contextual correlation projections, Proceedings of the Eighth European Conference on Computer Vision (2004) 562–573.
- [16] T. Melzer, M. Reiter, H. Bischof, Appearance models based on kernel canonical correlation analysis, Pattern Recognition 36 (9) (2003) 1961–1971.
- [17] Q.S. Sun, S.G. Zeng, Y. Liu, P.A. Heng, D.S. Xia, A new method of feature fusion and its application in image recognition, Pattern Recognition 38 (12) (2005) 2437–2448.
- [18] J.R. Kettenring, Canonical analysis of several sets of variables, Biometrika 58 (3) (1971) 433–451.
- [19] Y.O. Li, T. Adali, W. Wang, V.D. Calhoun, Joint blind source separation by multiset canonical correlation analysis, IEEE Transactions on Signal Processing 57 (10) (2009) 3918–3929.
- [20] J. Via, I. Santamaria, J. Perez, A learning algorithm for adaptive canonical correlation analysis of several data sets, Neural Network 20 (1) (2007) 139–152.
- [21] N.M. Correa, Y.O. Li, T. Adali, V.D. Calhoun, Fusion of FMRI, SMRI, and EEG data using canonical correlation analysis, IEEE International Conference on Acoustics, Speech and Signal Processing (2009) 385–388.
- [22] M.A. Hasan, On multi-set canonical correlation analysis, International Joint Conference on Neural Networks (2009) 1128–1133.
- [23] A.M. Martinez, R. Benavente, The AR face database, CVC Technique Report 24 (1998).
- [24] P.J. Phillips, P.J. Flynn, T. Scruggs, K.W. Bowyer, J. Chang, K. Hoffman, Marques, et al., Overview of the face recognition grand challenge, International Conference Computer Vision and Pattern Recognition (2005) 947–954.
- [25] F.X. Song, D. Zhang, D.Y. Mei, A multiple maximum scatter difference discriminant criterion for facial feature extraction, IEEE Transactions Systems Man and Cybernetics Part B: Cybernetics 37 (6) (2007) 1599–1606.